An Evolutionary Analysis of Insurance Markets with Adverse Selection

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The equilibrium nonexistence problem in Rothschild and Stiglitz’s insurance market is reexamined in a dynamic setting. Insurance firms are boundedly rational and offer menus of insurance contracts which are periodically revised: profitable competitors’ contracts are imitated and loss-making contracts are withdrawn. Occasionally, a firm experiments by withdrawing or innovating a random set of contracts. We show that Rothschild and Stiglitz’s candidate competitive equilibrium contracts constitute the unique long-run market outcome if innovation-experiments are restricted to contracts which are sufficiently “similar” to those currently on the market. Journal of Economic Literature Classification Numbers: C70, C72, D82, G22, L1.

Key Words: insurance markets, adverse selection, bounded rationality, imitation, local experiments, stochastic stability

* We thank Carlos Alós–Ferrer, Felix Höffler, Roman Inderst, Georg Nöldeke, Larry Samuelson, Richard Tunney, Fernando Vega–Redondo, two anonymous referees, and an associate editor for many helpful comments and suggestions.
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“Realized positive profits, not maximum profits, are the mark of success and viability. It does not matter through what process of reasoning or motivation such success was achieved. The fact of its accomplishment is sufficient. This is the criterion by which the economic system selects survivors: those who realize positive profits are the survivors; those who suffer losses disappear.”

Armen A. Alchian, (1950)

1. INTRODUCTION

Ever since Rothschild and Stiglitz’s (1976) seminal work on adverse selection in insurance markets, the problem of nonexistence of competitive equilibrium in screening markets has been a puzzle. Whenever Rothschild and Stiglitz’s (RS’s) competitive equilibrium exists, it consists of the pair of separating contracts which give the insurance takers the highest possible payoff under the condition that each contract makes zero profits. We call these the RS contracts. Often, RS’s competitive equilibrium does not exist because each firm has an incentive to deviate from offering an RS contract by offering a profitable pooling contract instead. The nonexistence problem becomes even more severe in the variant of the RS model where each firm may offer several contracts simultaneously; here, pairs of cross-subsidizing contracts can be profitable deviations, too (see Mas–Colell et al., 1995, p. 460).

Existing responses to RS tackle the nonexistence problem by changing the original equilibrium concept, the firms’ strategy spaces, or the sequential structure of the interaction. Dasgupta and Maskin (1986) show that a Nash equilibrium in mixed strategies exists. Wilson (1977) assumes that a deviating contract that would render some incumbent contracts loss-making, will be introduced only if it stays profitable after the withdrawal of the loss-making contracts. In this model, an equilibrium always exists; it consists of a single pooling contract whenever RS get nonexistence. However, even if one follows Wilson in assuming that firms anticipate the reactions of other firms before offering a deviating contract, it is still not clear why a firm could not offer a deviating contract and make profits until the other firms react. Miyazaki (1977) and Spence (1978) extend Wilson’s approach by allowing, in addition, for cross-subsidizing contracts. In these models, equilibria always exist, but may include loss-making contracts. Why does no firm withdraw the loss-making contract, letting the other firms carry the burden of cross-subsidization? Riley (1979) introduces the concept of reactive equilibrium. Here, it is the anticipation of further entry that deters firms from offering a deviating pooling contract, thus the RS contracts always constitute the unique equilibrium outcome. There exist a few models which vary the sequential structure of the interaction. In Grossman (1979), insurance takers first send a signal before firms make their contract offers, while in Hellwig (1987) firms can decline
to serve contracts after the insurance takers have made their choices. Although in both models equilibria exist, it is not clear whether the sequential structures reflect standard competitive markets.\textsuperscript{1}

A gap in this literature is that, in Wilson's (1977, p. 205) words, it lacks “an explicitly dynamic model, which describes how firms adjust their policies over time,” although many of the proposed equilibrium concepts are motivated by dynamic interpretations. Our model provides an explicitly dynamic solution to the equilibrium nonexistence problem, and thereby helps to evaluate some of the dynamic interpretations suggested in the literature.\textsuperscript{2}

In RS's insurance market,\textsuperscript{3} there are at least two risk neutral firms and a large population of risk averse individuals. Each individual is privately informed about her probability of accident, which may be high or low. Firms offer insurance contracts, each of which specifies a premium and an indemnity. Every individual buys a contract which maximizes her expected utility.

We study a dynamic version of RS's insurance market. At the beginning of each period, each firm offers a menu of one-period insurance contracts to the individuals. During the period, accidents occur and indemnities are paid accordingly. At the end of each period, the firms revise their menus according to the following boundedly rational rule. All competitors' contracts which made profits are imitated (i.e., added to the firm's menu), while all loss-making contracts are withdrawn. With some small probability, this imitation-and-withdrawal procedure is followed by an innovation-experiment, which adds a random set of contracts to the menu, or a withdrawal-experiment which removes a random set. Innovation-experiments are local in the sense that each experimental contract lies in a certain "similarity radius" of one of the currently offered contracts.\textsuperscript{4} (Our results continue to hold if experiments beyond the similarity radius are possible, but occur with much smaller probability than local experiments; cf. Section 4.)

The menu revision rule seems intuitively appealing. The core of the rule is imitation.\textsuperscript{5} Imitative behavior may be attractive to many decision makers because of its low informational and computational requirements. Specifically, imitation of contracts does not rely on information about the population's characteristics which may be difficult to obtain, such as risk types and their proportions, wealth levels,

\textsuperscript{1}See also Jaynes (1978), Hellwig (1988), Asheim and Nilssen (1996), and Inderst and Wambach (2001), for further variations of the sequential structure of the interaction.

\textsuperscript{2}Nödeke and Samuelson (1997) present a related dynamic model for signaling markets. In Section 5, we compare Nödeke and Samuelson's results to ours.

\textsuperscript{3}It is straightforward to reformulate the paper for Spence's (1973) job market (cf. Tröger, 1999), for Bester's credit market which is described by Hellwig (1987), and also for the standard price-discrimination setting with two consumer types (e.g., Fudenberg and Tirole, 1992, Section 7.1.1)).

\textsuperscript{4}This assumption is suggested by RS (1976, p. 646): “firms experiment with contracts similar to those already on the market.”

\textsuperscript{5}Behavior rules driven by imitation have been analyzed in various contexts; see Björnerstedt and Weibull (1996), Binmore and Samuelson (1997), Vega-Redondo (1997), Schlag (1998), and Gale and Rosenthal (1999), among others.
and utility functions. Instead it solely relies on market experience. Imitation also helps a firm to keep up with its competitors. Withdrawal of loss-making contracts appears also natural. As for random local experiments, these may be just “trembles” which result from imperfect imitation, occasional mispricing of contracts, or distorted communications inside the firm. Alternatively, such experiments may be a trial-and-error attempt of a firm with little information to increase its market share. Here, the local nature of experiments may reflect the incentives of the firm’s manager. If the comparison of the manager’s performance with that of her peers (or her own past performance) plays a role in evaluating the manager, then she might have an incentive to confine herself to local experiments. She might fear that the performance of a non-local experiment can differ too much from that of the contracts previously on the market, such that the failure of such an experiment would be disastrous for her evaluation.

The market dynamics resulting from the menu revision rule constitute a random (Markov) process. Any collection of firms’ menus is a possible state of the process, and the transition probabilities depend on the probabilities of the various experiments. In this process, the long-run relative frequency of each state is non-random and independent of the initial state. If the long-run relative frequency stays bounded away from zero for arbitrarily small experimentation probabilities then the state is called a long-run state (cf. Kandori et al., 1993, and Young, 1993). Assuming that experimentation probabilities are small, the market will almost always be in one of the long-run states. Therefore, our predicted market outcome consists of the contracts which are traded in long-run states.

Without experiments, any steady state of the imitation-and-withdrawal dynamics (“absorbing state”) would give rise to a possible market outcome. For example, any pair of profitable separating contracts would be a possible market outcome. Small-probability experimentation constitutes a perturbation which from time to time triggers a sequence of imitation and withdrawal steps from one absorbing state to another; those absorbing states which are most “robust” to this perturbation are the long-run states.

Our main findings are as follows. (a) All states in which the RS contracts are traded (“RS states”) are long-run states. I.e., the long-run prediction always includes the RS contracts, no matter which contracts are initially on the market, and independently of the similarity radius. In particular, neither cross-subsidizing nor pooling contracts can ever be the unique long-run prediction; they are always “dynamically unstable”. Cross-subsidizing contracts will successively become loss-making and therefore be withdrawn one after the other, while any pooling contract is upset by a local experiment. (b) Every long-run state is an RS state if (i) RS’s competitive equilibrium exists or (ii) the similarity radius is sufficiently small. I.e., whenever RS’s competitive equilibrium exists, it is the unique long-run prediction; if it does not exist, the RS contracts are still the unique long-run prediction if experimentation is sufficiently local. The latter prediction relies on the fact that firms are not able to find best replies; it could not have been obtained
with best-response dynamics. (c) If RS’s competitive equilibrium does not exist and the similarity radius is large (i.e., such that any two feasible contracts are considered similar), then all absorbing states are long-run states; in particular, the long-run prediction includes multiple outcomes.

The most important implication of these results is that there exists a unique long-run prediction – the RS contracts – if one assumes that the similarity radius is sufficiently small. This can be viewed as a dynamic solution to RS’s equilibrium nonexistence problem. This solution agrees with the Riley equilibrium, but differs from the other equilibrium concepts mentioned above. Our results make also clear that the long-run prediction depends crucially on the size of the similarity radius. In particular, if the similarity radius is large the equilibrium nonexistence problem persists in the sense that the long-run prediction includes multiple outcomes.

The rest of the paper is organized as follows. Section 2 specifies the model. Section 3 states and explains the main results. In Section 4, we discuss alternative menu revision rules. Section 5 compares our model with related evolutionary market models. Section 6 concludes. The Appendix contains the main proof.

2. THE MODEL

2.1. Rothschild and Stiglitz’s Insurance Market

Consider a countably infinite population of individuals facing the risk of losing \( L > 0 \) in an accident. Each individual has an initial wealth \( W \). There are two types of individuals: high risks in proportion \( \lambda \in (0, 1) \), who have a probability of accident \( \pi_h \), and low risks in proportion \( 1 - \lambda \) with a probability of accident \( \pi_l \in (0, \pi_h) \). Each individual is privately informed about her risk type. The average probability of an accident in the population is denoted \( \pi_{hl} = \lambda \pi_h + (1 - \lambda) \pi_l \).

There are \( n > 1 \) firms who may offer insurance to the individuals. An insurance contract \( c = (I, P) \) is characterized by an indemnity, \( I \), and a premium, \( P \). The insurance taker pays \( P \) to the firm, but gets paid \( I \) in case the accident occurs. Each firm \( j = 1, \ldots, n \) offers a set of insurance contracts \( S_j \), its menu. By \( s = (S_1, \ldots, S_n) \) we denote the collection of menus on the market. The set of all contracts on the market, \( \bigcup_{j=1}^n S_j \), will also be denoted by \( s \) (this abuse of notation should not cause confusion).

Each individual has to decide whether to sign one of the insurance contracts in \( s \) and which one. We assume that an individual’s utility from money is given by a twice continuously differentiable function \( u : \mathbb{R} \to \mathbb{R} \) with \( u' > 0 \) and \( u'' < 0 \). In particular, the individuals are risk averse. The payoff of an individual of type \( i = l, h \) who buys the insurance contract \( (I, P) \) is given by the expected utility

\[ U_i(I, P) = \pi_i u(W - P - L + I) + (1 - \pi_i) u(W - P). \]
Each individual maximizes this payoff. If there exists a contract \( c \in s \) with \( U_i(c) > U_i(0,0) \) then each individual of type \( i \) buys one of the contracts in
\[
\arg \max_{c \in s} U_i(c);
\]
otherwise, type \( i \) does not buy a contract. If a contract is offered by two or more firms, each individual buys from each firm with the same probability. We call \( c \in s \) active if at least one individual buys \( c \); otherwise \( c \) is called idle.

The firms are risk neutral. The expected profit per contract \((I, P)\) bought by an individual with expected probability of accident \( \pi \) is given by
\[ V((I, P), \pi) = \pi(P-I) + (1-\pi)P = P - \pi I. \]

Let \( \pi(c, s) \) be the average probability of accident in the set of individuals who buy \( c \in s \); if \( c \) is idle we do not define \( \pi(c, s) \). Then, any firm which offers \( c \) obtains the expected profit \( V(c, \pi(c, s)) \) per individual who buys \( c \). The sets of contracts which yield positive and negative expected profits are denoted
\[
B^+(s) = \{ c \in s \mid c \text{ is active, } V(c, \pi(c, s)) > 0 \},
\]
\[
B^-(s) = \{ c \in s \mid c \text{ is active, } V(c, \pi(c, s)) < 0 \}.
\]

Rothschild and Stiglitz (1976) present an analysis of this insurance market. As a solution concept, RS propose the competitive equilibrium set of contracts, which is defined by the following two conditions: first, each competitive equilibrium contract is active and makes nonnegative expected profits, and, second, there is no contract outside the equilibrium set that, if offered, would become active and make nonnegative expected profits.

RS identify a particular set of contracts, \( \{c^\text{RS}_l, c^\text{RS}_h\} \), as the unique candidate for a competitive equilibrium set (see Fig. 1). The high risks’ RS contract, \( c^\text{RS}_h = (I^\text{RS}_h, P^\text{RS}_h) \), is defined by \( I^\text{RS}_h = L \) and \( P^\text{RS}_h = \pi_h L \). The low risks’ RS contract, \( c^\text{RS}_l = (I^\text{RS}_l, P^\text{RS}_l) \), is defined by \( U_h(c^\text{RS}_h) = U_h(c^\text{RS}_l) \) and \( V(c^\text{RS}_l, \pi_l) = 0 \). Although being indifferent, all high risk individuals buy \( c^\text{RS}_h \). Note that the RS contracts can be characterized as the pair of separating contracts which give the individuals the highest possible payoff under the condition that the firms make zero profits with each contract.

RS point out that a competitive equilibrium set exists (if and) only if the set of profitable pooling deviations,
\[ \mathcal{P} = \{ c \in \mathbb{R}^2 \mid V(c, \pi_{hl}) > 0, U_l(c) > U_l(c^\text{RS}_l) \} \]
is empty. Indeed, if \( \mathcal{P} \neq \emptyset \) then \( \{c^\text{RS}_l, c^\text{RS}_h\} \) cannot be a competitive equilibrium set because any contract in \( \mathcal{P} \) attracts both risk types and makes nonnegative profits.

Our analysis will also make use of the Wilson (1977) pooling contract,
\[ c_{hl} = (I_{hl}, P_{hl}) = \arg \max_{c \in \mathbb{R}^2} \max_{V(c, \pi_{hl}) \geq 0} U_l(c), \]
which maximizes the low risks’ payoff among all contracts that yield nonnegative profits when sold to both types.

### 2.2. Discretization of the Contract Space

So far, we have assumed that any contract in \( \mathbb{R}^2 \) can be offered. However, for our dynamic analysis it is convenient to reduce the contract space to a discrete and finite grid with a (small) step size \( \delta > 0 \), a largest feasible indemnity, \( \bar{I} \), and a largest feasible premium, \( \bar{P} \). Precisely, we assume that only contracts in the set

\[
\Gamma^\delta \equiv \{ \delta, 2\delta, \ldots, \bar{I} \} \times \{ \delta, 2\delta, \ldots, \bar{P} \}
\]

are feasible. We also assume \( \bar{I} > I_{RS}^h \) and \( \bar{P} > P_{RS}^h \); i.e., \( \Gamma^\delta \) is large enough to contain a contract close to the high risks’ RS contract \( c_{RS}^h \) (the same is then also true for \( c_{RS}^l \)). Finally, we make several genericity assumptions.

**Assumption 2.1.** We have \( U_i(c) \neq U_i(c') \) and \( U_i(c) \neq U_i((0,0)) \) for \( c, c' \in \Gamma^\delta \) with \( c \neq c' \), and \( i = l, h \). Moreover, \( P/I \notin \{ \pi_l, \pi_h, \pi_{hl} \} \), for all \( (I,P) \in \Gamma^\delta \).

I.e., no individual is indifferent between different contracts, or between buying a contract and abstaining from the market; moreover, active contracts make either
profits or losses in expectation. An important implication of Assumption 2.1 is that in each state at most two contracts can be active.

The discrete competitive equilibrium set is defined by the same conditions as its continuous counterpart, except that the set of feasible contracts is $\Gamma^\delta$, instead of $\mathbb{R}^2$. The unique equilibrium candidate are the discrete RS contracts, $\{c^\delta_l, c^\delta_h\}$, defined by

$$U_h(c^\delta_h) = \max_{c \in \Gamma^\delta, V(c, \pi_h) > 0} U_h(c),$$

(1)

$$U_l(c^\delta_l) = \max_{c \in \Gamma^\delta, V(c, \pi_l) > 0, U_h(c) < U_h(c^\delta_h)} U_l(c).$$

(2)

These contracts constitute the discrete competitive equilibrium set if and only if the set of discrete profitable pooling deviations,

$$\mathcal{P}^\delta = \{c \in \Gamma^\delta | V(c, \pi_{hl}) > 0, U_l(c) > U_l(c^\delta_l)\},$$

is empty. Our analysis will also make use of the discrete Wilson pooling contract,

$$c^\delta_{hl} = \arg \max_{c \in \Gamma^\delta, V(c, \pi_{hl}) > 0} U_l(c).$$

The discrete model approximates its continuous counterpart in the following sense.\(^\text{6}\)

Remark 2.1. For $\delta \to 0$, we have $c^\delta_h \to c_{RS}^h$, $c^\delta_l \to c_{RS}^l$, $c^\delta_{hl} \to c_{hl}$, and $\mathcal{P}^\delta \to \mathcal{P}$.

In particular, if $\delta$ is sufficiently small, then $\mathcal{P}^\delta = \emptyset$ if and only if $\mathcal{P} = \emptyset$.

2.3. Stage Game

The firms’ interaction corresponds to the game in which the firms $j = 1, \ldots, n$ offer menus, $S_j \in \mathcal{M}^\delta \equiv \{S | S \subseteq \Gamma^\delta\}$, simultaneously and firms’ payoffs are determined by the anticipated choices of the individuals; we call this the stage game. The set of strategy profiles is denoted $\Omega^\delta \equiv \left(\mathcal{M}^\delta\right)^n$.

The discrete RS contracts are a Nash equilibrium outcome of the stage game if $\mathcal{P}^\delta = \emptyset$ and a certain “no cross subsidization” condition is fulfilled (cf. Mas–Colell \textit{et al.}, 1995, p. 465). On the other hand, if $\mathcal{P}^\delta \neq \emptyset$ and $\delta$ is small, then the discrete RS contracts are not a Nash equilibrium outcome (this is because some profitable pooling deviation is more profitable than both RS contracts together).

2.4. Similar Contracts

Our menu revision rule will assume that firms experiment “with contracts similar to those already on the market” (RS, 1976, p. 646). It is convenient to define

\(^6\)We use the standard topology to define convergence of contracts; i.e., $(\Gamma^\delta, \mathcal{P}^\delta) \to (I, P)$ if and only if $I^\delta \to I$ and $\mathcal{P}^\delta \to \mathcal{P}$. The Hausdorff distance topology is used for convergence of sets, $\mathcal{P}^\delta \to \mathcal{P}$.
similarity on \( \mathbb{R}^2 \); fix a norm \( || \cdot || \) on that space,\(^7\) and a similarity radius \( r > 0 \). Contracts \( c, c' \in \mathbb{R}^2 \) are called \( r \)-similar if \( ||c - c'|| < r \). We define the \( r \)-neighborhood of a set of contracts \( S \subseteq \mathbb{R}^2 \) with respect to the discrete contract space; i.e.,
\[
\mathcal{N}_r^\delta(S) = \{ c' \in \Gamma^\delta \mid \exists c \in S, \ c \text{ is } r \text{-similar to } c' \}.
\]
By \( r_0 \) we denote the largest similarity radius such that no profitable pooling deviation is similar to a contract that the high risks do not prefer to their RS contract; i.e., if \( \mathcal{P} \neq \emptyset \) let
\[
r_0 = \inf_{d \in \mathbb{R}^2, \ U_h(d) \leq U_h(c^{R_S}_h), \ c \in \mathcal{P}} ||c - d||;
\]
if \( \mathcal{P} = \emptyset \), let \( r_0 = \infty \). Note that \( r_0 > 0 \). Let us now define an equivalent to \( r_0 \) in the discrete contract space. If \( \mathcal{P}^\delta \neq \emptyset \), let
\[
r_0^\delta = \min_{d \in \Gamma^\delta, \ U_h(d) \leq U_h(c^{\delta}_h), \ c \in \mathcal{P}^\delta} ||c - d||;
\]
if \( \mathcal{P}^\delta = \emptyset \), let \( r_0^\delta = \infty \). Remark 2.1 implies \( r_0^\delta \to r_0 \) as \( \delta \to 0 \). Therefore:

Remark 2.2. Let \( r < r_0 \). Then, \( r < r_0^\delta \) if \( \delta \) is sufficiently small.

Note that if \( r < r_0^\delta \) then no discrete profitable pooling deviation is \( r \)-similar to a discrete RS contract. Hence, Remark 2.2 implies that if \( r < r_0 \) and the step size \( \delta \) is sufficiently small and each firm’s menu consists of the discrete RS contracts, then there exists no profitable single-contract deviation in the \( r \)-neighborhood of this menu. With this in mind, the RS contracts may be called a “local equilibrium” outcome of the stage game (cf. RS, 1976, p. 646).

2.5. Dynamic Behavior of the Firms

Suppose the insurance market described above opens every period \( t = 1, 2, \ldots \). Our goal is to determine which contracts are active most frequently in the long run if menus are periodically revised according to a rule based on imitation, withdrawal, and occasional experiments.

Any collection of menus, \( s \in \Omega^\delta \), is called a state. We call a state separating if two different contracts are active, both make profits, and both are offered by all firms. Note that in a separating state each firm may offer any set of idle contracts in addition to the active contracts. A separating state is called an RS state if the discrete RS contracts are the active ones. i.e., \( s = (S_1, \ldots, S_n) \in \Omega^\delta \) is an RS state if (i) \( \{c^{\delta}_l, c^{\delta}_h\} \subseteq (S_1 \cap \ldots \cap S_n) \), and (ii) all contracts in \( s \setminus \{c^{\delta}_l, c^{\delta}_h\} \) are

\(^7\)Our results hold for all norms because of the well-known “equivalence” of norms on \( \mathbb{R}^2 \). Any two norms \( || \cdot ||_1, || \cdot ||_2 \) on \( \mathbb{R}^2 \) are equivalent in the sense that there exist \( \alpha > 0 \) and \( \beta > 0 \) such that, for all \( c \in \mathbb{R}^2 \), \( \alpha ||c||_2 \leq ||c||_1 \leq \beta ||c||_2 \).
idle. A state is called pooling if there exists a single active contract, the active contract is offered by all firms, is bought by both risk types, and makes profits. A high-risks-only state is like a pooling state except that the active contract is bought by the high risks only, while the low risks do not buy any contract. Idle states are those in which all contracts on offer are idle. A particular idle state is the dead-market state in which no contract is on offer. States which involve loss-making contracts belong to none of these categories. There exist separating, pooling, and high-risks-only states with lots of different active contracts.

**Imitation and Withdrawal**

Suppose, at the beginning of period \( t \) the market is in state

\[
s(t) = (S_1(t), \ldots, S_n(t)).
\]

After contracts in \( s(t) \) have been traded, accidents have occurred, and profits have been realized, each firm revises its menu. On making this decision, the firms take into account the observed profits of the contracts in \( s(t) \).

**Assumption 2.2.** At the end of each period, each firm observes which contracts made positive profits, and which contracts in its own menu made losses.

Given these observations, firms add all profitable contracts to their menus, while withdrawing all loss-making contracts. I.e., firm \( j \) revises its menu \( S_j(t) \) to the menu

\[
S'_j(t) = (S_j(t) \cup B^+(s(t))) \setminus B^-(s(t)).
\]

**Local Experimentation**

After imitation and withdrawal have taken place, each firm experiments with a certain probability \( \epsilon < 1 \) independent across firms and across time. For each firm \( j \), an experiment is either the withdrawal of a random subset of (idle and/or active) contracts from its current menu, \( S'_j(t) \), or the innovation of a random set of contracts, each of which is \( r \)-similar to a contract in \( s(t) \). Moreover, firms may always experiment with small-premium-small-indemnity contracts; i.e., with contracts that are \( r \)-similar to \((0, 0)\). Hence, firm \( j \)'s menu \( S_j(t+1) \) at the beginning of period \( t+1 \) has the following properties.

\[
\begin{align*}
S_j(t + 1) &= S'_j(t) & \text{if } j \text{ does not experiment,} \\
S_j(t + 1) &\subseteq S'_j(t) & \text{if } j \text{ makes a withdrawal-experiment,} \\
S'_j(t) &\subseteq S_j(t + 1) \subseteq \mathcal{N}_r^x(s(t) \cup \{(0,0)\}) & \text{if } j \text{ makes an innovation-experiment.}
\end{align*}
\]

\(^8\) Since the population is countably infinite, by the strong law of large numbers, with probability 1 the set of contracts with positive (resp. negative) realized profits equals \( B^+(s(t)) \) (resp. \( B^-(s(t)) \)). In Section 4 we further discuss the role of the population size.
The smaller the similarity radius \( r \) is, the more "local" the firms' experiments might be called. If \( r \) is sufficiently large, experimental contracts can be anywhere on the contract space; for such \( r \), we say that experimenting is global.

The probability distribution over experiments is assumed to be independent of the period number \( t \) (it depends, however, on the current state \( s(t) \)). The exact specification of this distribution will turn out to be irrelevant for the results, up to the following restrictions.

**Assumption 2.3.** Each of the following experiments occurs with positive probability for every firm \( j \): withdrawal of any single contract in \( S'_j(t) \), and innovation of any single contract in \( N^\delta_r(S'_j(t) \cup \{(0,0)\}) \).

In particular, no firm is required to innovate a contract that is similar only to a loss-making contract or a competitor’s contract, nor is any firm required to innovate several contracts simultaneously. Finally, we assume the step size \( \delta \) is so small that starting from any state (even the dead-market state), an innovation-experimental contract exists.

**Assumption 2.4.** We have \( |(0,\delta)| < r \), \( |(\delta,0)| < r \), and \( |(\delta,\delta)| < r \).

**Markov Process**

Imitation, withdrawal and experimentation induce the transition from state \( s(t) \) at the beginning of period \( t \) to state \( s(t+1) = (S_1(t+1), \ldots, S_n(t+1)) \) at the beginning of period \( t+1 \). For every pair of states \( s, s' \in \Omega^\delta \) the probability of transition from \( s \) to \( s' \) is denoted \( P^{\epsilon,\delta,r}_{s,s'} \). Note that, by construction, the transition probabilities do not depend directly on \( t \). The Markov process \( P^{\epsilon,\delta,r} = (P^{\epsilon,\delta,r}(s,s'))_{s,s' \in \Omega^\delta} \) is called the evolutionary insurance market (with experimentation rate \( \epsilon \), step size \( \delta \), and similarity radius \( r \)).

A state \( s \) is called absorbing if \( P^{0,\delta}_r(s,s) = 1 \); i.e., the absorbing states are the steady states of the market without experiments (\( \epsilon = 0 \)). Proposition 2.1 characterizes the absorbing states and shows that the market without experiments will end up in an absorbing state; before it ends up there, the set of contracts on the market may shrink, but cannot increase. The RS states are absorbing, but many other states are, too.

**Proposition 2.1.** A state is absorbing if and only if it is a separating, pooling, high-risks-only or idle state. Starting from any state \( s(t) \), the market \( P^{0,\delta}_r \) reaches an absorbing state \( s(t') \) with \( s(t') \subseteq s(t) \) in some period \( t' \geq t \).

**Proof.** We start with the characterization of the absorbing states. The “if” part is immediate from the definition of imitation and withdrawal. For the “only if” part it is sufficient to show that, starting from any state in period \( t \), by imitation and withdrawal a separating, pooling, high-risks-only or idle state is reached. Note
that the set of contracts on the market cannot increase,
\[ s(t) \supseteq s(t+1) \supseteq s(t+2) \supseteq \ldots. \]
Hence, there exists a period \( t'' \) such that
\[ s(t'') = s(t'' + 1) = s(t'' + 2) = \ldots. \]
By construction, in state \( s(t'') \) no firm offers a loss-making contract. Therefore \( s(t') \) \((t' \equiv t'' + 1)\) is a separating, pooling, high-risks-only or idle state.

The proof relies on the fact that without experiments no innovations can occur; after some periods an absorbing state is reached because all loss-making contracts have been withdrawn, and all possibilities of imitation have been exhausted. The market is then in a separating, pooling, high-risks-only or idle state, and all such states are trivially absorbing.

The properties of the market with experiments \((\epsilon > 0)\) are very different from those of the market without experiments. In particular, no steady state exists because every state can be left by an experiment. The following remark shows that the long-run properties of the market are captured by a single probability distribution which is independent of the initial state. This is a straightforward implication of standard results (see Freidlin and Wentzell, 1984, Fudenberg and Levine, 1998, p. 170).

\textbf{Remark 2.3.} Consider a process \( P = P^\epsilon,\delta r \) with \( \epsilon > 0 \). There exists a probability distribution \( \mu^\epsilon,\delta r \) over \( \Omega^\delta r \) such that
(i) \( \mu^\epsilon,\delta r \) is the (unique) invariant distribution (i.e., \( P \cdot \mu^\epsilon,\delta r = \mu^\epsilon,\delta r \)),
(ii) \( \mu^\epsilon,\delta r = \lim_{T \to \infty} P^T \cdot \mu_0 \), for every initial distribution \( \mu_0 \),
(iii) for all initial states and with probability 1, the relative frequency of state \( s \in \Omega^\delta r \) in the first \( T \) periods, converges to \( \mu^\epsilon,\delta r(s) \) for \( T \to \infty \).

The limit distribution \( \mu^\epsilon r = \lim_{\epsilon \not= 0, \epsilon \to 0} \mu^\epsilon,\delta r \) exists. If \( s \in \Omega^\delta r \) and \( \mu^\epsilon r(s) > 0 \), then \( s \) is absorbing.

A state \( s \in \Omega^\delta r \) is called a \textit{long-run} state if \( \mu^\epsilon r(s) > 0 \) (see Kandori et al., 1993, and Young, 1993). Note that only absorbing states can be long-run states. Assuming that the experimentation rate \( \epsilon \) is small, (ii) implies that no matter in which state the market is initially, after a long time it will most probably be in a long-run state, and (iii) implies that if one looks at the market for a long stretch of time, one will almost always observe a long-run state. Due to (ii) and (iii), the contracts which are active in long-run states may be predicted as the long-run market outcome.

In particular, ergodicity holds because from any state, the dead-market state can be reached with positive probability, by simultaneous withdrawal-experiments of all firms, and once in the dead-market state, there is positive probability of remaining there for one period.
3. CHARACTERIZATION OF THE LONG-RUN STATES

In this section, we determine the long-run states of the evolutionary insurance market. We find that (a) the RS states are long-run states for any similarity radius. Moreover, (b) the RS states are the only long-run states if RS's competitive equilibrium exists or the similarity radius is small. (c) If RS's competitive equilibrium does not exist and experimentation is global, every absorbing state is a long-run state. All results require that the step size $\delta$ is small. The proof is relegated to the Appendix.

**Proposition 3.1.** For all $r > 0$, there exists $\delta(r) > 0$ such that for all $\delta < \delta(r),$

(a) every RS state is a long-run state,
(b) if $P^{\delta} = \emptyset$ or $r < r_0$ then every long-run state is an RS state,
(c) if $P^{\delta} \neq \emptyset$ and $r$ is sufficiently large then all separating, pooling, high-risks-only and idle states are long-run states.

The most important implication is obtained from (a) and (b): there exists a unique long-run prediction – the discrete RS contracts – if one assumes that the similarity radius is sufficiently small ($r < r_0$). This uniqueness result can be viewed as a dynamic solution to RS's equilibrium nonexistence problem. It holds even if the RS states do not correspond to Nash equilibria of the stage game. If, however, one cannot assume a small similarity radius, the uniqueness result breaks down and the set of long-run outcomes is rather large, as shown in (c).

In the following we outline the market dynamics which underly Proposition 3.1. Throughout the outline we assume that the step size is sufficiently small; the qualification “discrete” is left out everywhere. We call an experiment isolated if (i) it is made in an absorbing state and (ii) after the experiment imitation and withdrawal will bring the market back to an absorbing state before a further experiment occurs.

To get (a), we show that starting from any absorbing state, there is some sequence of isolated experiments which brings the market to an RS state. This is sufficient because the small probability of experiments implies that they will almost always be isolated. We distinguish four cases (see Fig. 2).

**Starting in an absorbing state where the high risks buy their RS contract.** (“A”)

We construct a finite sequence of contracts for the low risks, each contract $r$-similar to the previous one, which ends with the low risks’ RS contract. All the contracts in the constructed sequence can be introduced by successive isolated experiments. Each experimental contract will be imitated. The sequence starts with the contract which the low risks are buying in the current state; if the low

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10 It seems also worth mentioning that the expected waiting time until the market first reaches a long-run state is small, namely of order $\epsilon^{-1}$; this follows from Ellison (2000).
FIG. 2. The diagram indicates how starting from any absorbing state, an RS state can be reached by a sequence of isolated experiments. Every absorbing state is an element of one of the sets $A, B, C, D$. The set $A$ includes the RS states. Arrows indicate sequences of isolated experiments.

risks are currently buying no insurance, the sequence starts with a contract that is $r$-similar to $(0, 0)$ and provides a bit insurance, such that it attracts the low risks only and makes profits. The next contract in the sequence is constructed such that it is profitable and attracts the low risks away from the first contract because it provides a bit more insurance or is cheaper. In this way, the construction is continued until a contract is reached such that any other contract that is yet more attractive to the low risks, while not attracting the high risks, would makes losses; at this point, the low risks’ RS contract has been reached, and the market is in an RS state.

Starting in an absorbing state where the high risks do not buy their RS contract, and the contract they buy would stay profitable if the low risks left the market. ("B")

There exists an experimental contract which attracts the high risks (it may also attract the low risks) and would stay profitable if the low risks left the market. If the new contract is the high risks’ RS contract, an absorbing state of class $A$ above is reached by imitation. Otherwise, the new absorbing state belongs to class $B$ again, but the payoff of the high risks has increased in this “loop.”. After finitely

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11That such a contract exists is obvious if the current contract is not close to the high-risk zero-profit line; the experiment consists then in a slight reduction of the premium. Suppose now the current contract is close to this zero-profit line. If the high risks are about fully insured (i.e., if the current indemnity is about $L$), then the current contract is similar to the high risks’ RS contract, and thus the latter may occur by an experiment. If the high risks are currently underinsured, there exists an experiment which provides a bit more insurance while keeping about constant the profit which is obtained from the high risks; the overinsurance case is treated analogously.
many such loops, class \( A \) will be reached because the range of feasible payoff levels is finite.

**Starting in a pooling state where the active contract would make losses if the low risks left the market. (“C”)**

First, all idle contracts are removed from the market by a sequence of isolated withdrawal-experiments. Let \( c = (I, P) \) be the active contract. We may construct a contract \( c' = (I', P') \) which is \( r \)-similar to \( c \), offers less insurance \((I' < I)\) and has a reduced premium \((P' < P)\). Because the high risks require a larger reduction of premium than the low risks to accept less insurance, \( c' \) can be chosen such that it attracts the low risks only. Moreover, choosing \( c' \) sufficiently close to \( c \) assures that \( c' \) is profitable. A firm which experiments with \( c' \), the incumbent contract \( c \) makes losses because only the high risks buy it. Consequently, all firms withdraw \( c \) while imitating \( c' \). Then \( c' \) becomes the unique active contract, and it is bought by everybody.

Suppose first that \( c' \) now makes losses. In this case, it will be withdrawn and the dead-market state is reached. I.e., we have described an isolated experiment which leads to an idle state (class \( D \)). Second, suppose that \( c' \) is still profitable. In this case, we have described an isolated experiment which leads to class \( B \), or again to a state of class \( C \), but the payoff of the low risks is increased in this “loop.” Therefore, eventually class \( B \) or \( D \) is reached.

**Starting in an idle state. (“D”)**

The \( r \)-neighborhood of the contract \((0, 0)\) contains a pair of separating profitable contracts (this holds because each individual is risk averse, and the types differ by their probabilities of accident). Suppose, a firm makes an isolated experiment with the contract which is designed for the high risks. This contract makes profits (even if the low risks buy it, too). Therefore, the experiment is followed by imitation. Now the contract which is designed for the low risks may be introduced by another isolated experiment. The result is a separating state (class \( A \) or \( B \)). This completes the proof that an RS state can be reached from anywhere by a sequence of isolated experiments.

To get (b), it is sufficient to show that the RS states are stable with respect to all isolated experiments if \( P^\delta = \emptyset \) or \( r < r_0 \). By stability we mean that after any isolated experiment made in an RS state imitation and withdrawal will bring the market back to a, possibly different, RS state.

Stability is most obvious for withdrawal-experiments: after one period, the experimenting firm will imitate from the other firms whatever RS contract it had withdrawn. Let us turn to innovation-experiments. We can confine ourself to ex-
periments with at least one profitable contract because in all other cases withdrawal will bring the market back to an RS state within one period.

**Stability with respect to an innovation-experiment with a single contract.**

RS’s (1976) original arguments show that to be profitable, an experimental contract must be a profitable pooling deviation; i.e., must be taken from $P^δ$. However, if a profitable pooling deviation exists ($P^δ \neq \emptyset$) then it cannot be introduced by an isolated experiment because the similarity radius is too small ($r < r_0$).

**Stability with respect to an innovation-experiment with a pair of cross-subsidizing contracts.**

In the spirit of Miyazaki (1977) and Spence (1978), a firm might innovate two contracts $c$ and $c'$ simultaneously: $c$ attracts the low risks because it offers a bit more insurance than the RS contract $c^δ_l$, and $c'$ attracts the high risks because it has a lower premium than the RS contracts $c^δ_h$. While $c'$ obviously makes losses, it is useful for distracting the high risks from $c$ so that $c$ attracts the low risks only and makes profits; i.e., $c$ and $c'$ “cross-subsidize” each other. If the proportion of low risks in the population is large enough, the overall profit of $c$ and $c'$ can be positive.

However, in the third period after the pair of cross-subsidizing contracts has occurred, the market will have returned to an RS state. In the first period after the experiment, all firms will imitate the low risks’ contract $c$, while still offering the (idle) RS contracts. At the same time, the experimenting firm will withdraw $c'$. Therefore, two periods after the experiment, $c$ will attract both types and thus will make losses. Consequently, at the end of the period $c$ will be withdrawn such that the market is again in an RS state.

No new issues arise from innovation-experiments with three or more contracts: at least one experimental contract will make losses, and after it is withdrawn another experimental contract will start making losses so that the market eventually returns to an RS state.

To get (c), it is sufficient to construct a sequence of isolated experiments from the RS state without idle contracts to any absorbing state. The construction begins with an innovation-experiment with the Wilson pooling contract. After it is imitated, the RS contracts are removed by a sequence of withdrawal-experiments. Now a cream-skimming experiment leads to the dead-market state. From there, any absorbing state with a unique active contract is reached by an innovation-experiment with just this contract, followed by a sequence of innovation-experiments with idle contracts. To reach a separating state instead, it is sufficient that an isolated experiment with the high-risks’ contract is followed by an isolated experiment with the low risks’ contract, and this is followed by the innovation of any idle contracts.
4. SOME REMARKS ABOUT THE MENU REVISION RULE

Items 1–4 discuss alternative imitation-and-withdrawal rules, items 5–8 discuss alternative experimentation rules, item 9 highlights the role of the population size, and item 10 is about best-response dynamics.

1. The assumption that each period all firms imitate all profitable contracts can be weakened. First of all, inertia (cf. Kandori et al., 1993) could be introduced, in the sense that each period each firm with some probability smaller than 1 does not imitate. Secondly, it is not crucial that an imitating firm always adds all profitable contracts to its menu; rather, each profitable contract must be added with positive probability which may, e.g., depend on the contract’s profitability or on the number of firms offering this contract. If, however, all firms are so cautious that each period only the most profitable contract is imitated, and innovation-experiments are restricted to contracts similar to those in the experimenter’s own menu (rather than to any current market contract), the RS contracts are in some cases not the unique long-run outcome even if the similarity radius is arbitrarily small,\(^{12}\) in contrast to Proposition 3.1 (b).

2. According to Assumption 2.2, each firm is able to identify the set of profitable contracts on the market. Alternatively, we may assume that firms only observe competitors’ menus and total profits, while the profitability of individual contracts is not generally observable. Under this assumption, our imitation rule is not feasible, but firms might still imitate the contracts of the most successful firm and withdraw loss-making contracts, as follows. Denote by \(J^*(t)\) the set of firms which have maximum profits among all firms. Any firm \(j \notin J^*(t)\) changes its menu to

\[ S'_j(t) = (S_j(t) \cup S_{j^*}(t)) \setminus \left( B^{-}(s(t)) \cap S_j(t) \right), \]

with any \(j^* \in J^*(t)\); any firm \(j \in J^*(t)\) changes its menu to

\[ S'_j(t) = S_j(t) \setminus B^{-}(s(t)). \]

This rule yields essentially the same market dynamics as the rule analyzed in the main text.

3. We have defined imitation such that profitable contracts of competitors are added to the imitator’s menu, while existing profitable contracts in the imitator’s menu are retained. Alternatively, imitation could be defined as the substitution of the imitator’s existing menu for a more profitable competitor’s menu. That would be closer to the existing literature on imitation (see footnote 5 and Section 5),

\(^{12}\)Consider, for instance, the case where the high risks’ discrete RS contract would be less profitable than the low risks’ even if the former were offered by one firm and the latter by all. In this case, the dead-market state is a long-run state because starting from the RS state without idle contracts, a sequence of isolated withdrawal-experiments removes the high risks’ contract from the market, implying that the low risks’ contract makes losses and the dead-market state is reached by withdrawal.
where imitation is usually defined as the substitution of one’s strategy by that of someone else with higher payoff. No imitation rule of that kind, however, makes the RS contracts the unique long-run outcome, no matter how small the similarity radius is. This is because, in contrast to the discussion on p. 16, the RS states are not stable with respect to cross-subsidizing experiments. To restore stability, one has to assume that no experiment innovates more than one contract.\footnote{We conjecture that this assumption restores all our results if imitation is based on substitution with additional conditions like those introduced in Alós-Ferrer et al. (2000).}

4. We have assumed that idle contracts are not withdrawn except via an experiment. A maybe more plausible alternative assumption would be that firms withdraw all contracts that stay idle for a fixed number $m$ of consecutive periods.\footnote{The state space of the underlying Markov process must be augmented in order to take account of such a rule; i.e., the state at the beginning of period $t$ includes the current collection of menus as well as the collections of menus offered during the periods $t - m + 1, \ldots, t - 1$. As a consequence, a state is absorbing only if all firms offer the same menu $S$ for $m$ consecutive periods and no contract in $S$ is idle.} In such a market, Proposition 3.1 (a) and (c) hold (with appropriately adapted definitions of the various types of absorbing states). However, it is easy to see that (b) fails if $m = 1$; i.e., our solution to the equilibrium nonexistence problem breaks down if idle contacts are withdrawn very quickly.\footnote{To see this, suppose the market is in an RS state. If one firm experiments, for example, with a reduction of premium of the high risks’ RS contract, the experimental contract will make losses while the high risks’ RS contract becomes idle. Hence, at the end of the period only the low risks’ RS contract will remain on the market. In the subsequent period, this contract will make losses, too, because it attracts all individuals. Therefore, the dead-market state is reached by an isolated experiment. Together with Proposition 3.1 (a), this argument shows that if $m = 1$ the dead-market state is a long-run state for any similarity radius, in contrast to Proposition 3.1 (b).} Proposition 3.1 (b) goes through if the number of new contracts in innovation-experiments is bounded above and $m$ is sufficiently large. In other words, our solution to the equilibrium nonexistence problem is restored if firms are conservative both in the sense that they do not innovate too many contracts at once, and that they do not withdraw idle contracts too quickly. For example, $m \geq 3$ is sufficient to restore Proposition 3.1 (b) if innovation-experiments do not involve more than two contracts. To see this, suppose that at the end of a period where the market is in an RS state, some innovation-experiment introduces up to two new contracts to the market. Arguments like those on p. 16 show that each of the new contracts will become loss-making, and thus be withdrawn, within at most two periods. Therefore, the RS contracts are idle for at most two periods, and thus will not be withdrawn if $m \geq 3$.

5. We have assumed that no firm ever experiments with a contract that is not $r$-similar to one currently on the market or to $(0, 0)$. This assumption can be weakened: global experiments may occur if their probability is much lower than that of local experiments. More precisely, we may allow that with probability $\epsilon^2$ a firm makes a global experiment (the probability of no experiment is then $1 - \epsilon - \epsilon^2$, so we add the assumption $\epsilon < 1/2$). Our results continue to hold because
any sequence of isolated experiments has a much higher probability than a global experiment if experiments are rare. Put differently, after any global experiment, the market will usually reach an RS state long before the next global experiment occurs.

6. According to Assumption 2.3, withdrawal-experiments may include profitable contracts. This seems implausible if experiments are trial-and-error attempts to increase market share, as we suggested in the Introduction. Fortunately, such experiments are not needed anywhere except in the proof of Remark 2.3 (see footnote 9). To restore Remark 2.3, it is sufficient to add the assumption of a sufficiently small step size. To see this, note that the proof of Proposition 3.1 (a) shows that the RS state without idle contracts can be reached from any state with positive probability if $\delta$ is sufficiently small; hence, this RS state, instead of the dead-market state, can be used in footnote 9. Although withdrawal-experiments with profitable contracts are not essential, we have allowed for such experiments in order to make clear that, if isolated, they cannot destabilize the RS states (cf. Proposition 3.1 (b)).

7. Our definition of similarity allows innovation-experiments with contracts which neither underbid the premium nor overbid the indemnity, of any of the contracts previously on the market. Such experiments will obviously not attract any individual (unless other contracts are withdrawn). Therefore, such experiments appear implausible in the context of our suggestion in the Introduction that experiments are trial-and-error attempts to increase market share. Because the only step where we use these implausible experiments is in the proof of Lemma A.3, none of our predictions about long-run market outcomes would change without these experiments; in particular, Proposition 3.1 would go through if (a) were changed to: “the RS state without idle contracts is a long-run state.”

8. Innovation-experiments with small-premium-small-indemnity contracts are essential to Proposition 3.1. Without such experiments, the dead-market state is the unique long-run state, because it is reached with positive probability and cannot be left.

9. Our assumption that the population is countably infinite simplifies the dynamics because it implies that realized profits equal expected profits (cf. footnote 8). If the population is finite, the dynamics depend on the size of the population, $N$, relative to the experimentation rate, $\epsilon$. Our results hold if, for any given $\epsilon$, $N$ is large. In fact, the weak law of large numbers implies that if $N$ is sufficiently large the probability that realized and expected profits differ significantly is arbitrarily small. Let $P_{r}^{N,\epsilon,\delta}$ be the Markov process of the insurance market with population size $N$; let $\mu_{r}^{N,\epsilon,\delta}$ be the respective invariant distribution. Then, $P_{r}^{\epsilon,\delta} = \lim_{N \to \infty} P_{r}^{N,\epsilon,\delta}$, which implies that $\mu_{r}^{\epsilon,\delta} = \lim_{\epsilon \to 0} \lim_{N \to \infty} \mu_{r}^{N,\epsilon,\delta}$.

It is easy to see that our results do not go through if the order of limits is reversed; i.e., if, for any given $N$, $\epsilon$ is small. Indeed, any contract $(I, P)$ with $I > P$ can make losses because everybody in the whole population may have
an accident. Moreover, individuals never buy contracts with \( I \leq P \). Thus, in every feasible state there is positive probability that all active contracts are withdrawn without an experiment. Therefore, for any fixed population size, the unique long-run prediction is that no contracts are traded. In particular, the distribution \( \lim_{N \to \infty} \lim_{\epsilon \to 0} \mu_{r}^{N,\epsilon,\delta} \) puts all its weight on the idle states.

10. An important alternative to our menu revision rule is best-response, where firms play myopic best replies, with or without inertia, to the competitors’ previous-period menus. Assume for simplicity that firms never offer contracts they expect to be idle. Best-response dynamics are very different from our dynamics. First, any absorbing state corresponds to a Nash equilibrium of the stage game. This situation differs remarkably from the vast multiplicity of non-equilibrium absorbing states identified in Proposition 2.1. Second, if RS’s competitive equilibrium does not exist and \( \delta \) is small then even the RS state without idle contracts fails to be a Nash equilibrium of the stage game and thus also fails to be an absorbing state; in this case, the discrete RS contracts cannot be the unique market outcome, in contrast to Proposition 3.1.

In Section 5, we discuss Nöldeke and Samuelson’s (1997) perturbed best-response dynamics for signaling games, adapted to the insurance market. These dynamics are very different from the best-response dynamics discussed above. In particular, NS’s model predicts a vast multiplicity of absorbing states, each of which corresponds to a sequential equilibrium of the underlying signaling game.

5. RELATED EVOLUTIONARY MARKETS

In this section we review related evolutionary papers. Our menu revision rule is inspired by those in Vega-Redondo (1997) and Alós-Ferrer et al. (1999). Nöldeke and Samuelson (1997) analyze evolutionary dynamics in signaling markets.

Vega-Redondo (1997) studies evolution in Cournot markets. There, each firm has a one-dimensional decision variable, quantity. Hence, the firms’ strategy space is quite different from that of the insurance market where each firm may offer any finite menu of two-dimensional contracts. Vega-Redondo shows that imitation and experimentation lead to the Walrasian market outcome. That is, as in our case, the “competitive” outcome is selected. In Vega-Redondo, this selection is due to the effect of spite—a firm that experiments with the Walrasian quantity may hurt itself in terms of profits, but it will hurt its competitors even more. However, in

\[16\]

In addition to absorbing states, there may exist nonsingleton absorbing sets which represent possible market outcomes. Characterizing all absorbing sets appears complicated. It is not even clear whether the RS state without idle contracts is always contained in an absorbing set. A sufficient condition would be that, starting from any state, some sequence of transitions leads to the RS state. This condition would imply the existence of a unique absorbing set containing the RS state. In case of nonexistence of RS’s competitive equilibrium, the absorbing set would also contain pooling states. Furthermore, the absorbing set would be identical to the set of long-run states of the best-response dynamics with random experimentation, whether local or global.
our case, the competitive outcome is selected because by undercutting premium
or increasing indemnity, customers are attracted and competitors are hurt, as in
Bertrand competition.\footnote{Alós-Ferrer \emph{et al.} (2000) analyze an evolutionary model based on imitation for the Bertrand market.}

Alós-Ferrer \emph{et al.} (1999) analyze a model closely related to the one in Vega-
Redondo (1997), but in a context where returns to scale may be increasing – the
Walrasian equilibrium may not exist – and with explicit entry and exit of firms.
Behavior there is based on imitation and local experimentation. For a fixed number
of firms, it is shown that due to the effect of spite the dynamics lead to the quantity
corresponding to a symmetric marginal cost-pricing equilibrium as defined in the
theory of general equilibrium with non-convex technologies.\footnote{See Alós-Ferrer \emph{et al.} (1999) for the exact definition.} For convex costs,
this corresponds to the Walrasian equilibrium.

Nöldeke and Samuelson (1997) present an evolutionary model for a wide range
of signaling markets.\footnote{A related model is Jacobsen \emph{et al.} (2001).} Adapted to RS’s insurance market – a screening market
– NS’s model reads as follows. The firms are represented by a single firm. Its
menu includes exactly one contract per feasible indemnity. The firm has a belief
about the proportion of insurance takers of each risk type who will choose each
contract. Premia are calculated such that, given the beliefs, the expected profits
with each contract are zero; i.e., the firm prices competitively. The underlying
idea is that the firms on the market are in Bertrand competition for each indemnity,
so all firms offer an identical menu: the menu of the representative firm. The state
of the market is given by the beliefs (or, equivalently, the current menu). Beliefs
are updated according to the previous period’s market experience, but occasionally
an off-play-path belief mutates randomly. Via the zero-profit condition, updates
and mutations, in effect, change premia. In particular, a mutation may result in
any premium that can be rationalized by a belief.

NS’s model differs from ours in several respects. First, NS do not introduce a
concept of local mutation. Thus, their model lacks anything similar to our local
experiments. Second, NS’s model is best-response based, while ours is imitation
based. This implies quite different assumptions in terms of what firms observe.
In order to follow NS’s menu revision rule, firms must observe the population’s
characteristics (risk types and their proportions, wealth levels, and utility func-
tions), while our menu revision rule is independent of these characteristics. On
the other hand, our menu revision rule requires that firms observe profits of com-
petitors.\footnote{This raises the idea that, depending on what firms observe, they might use our menu revision rule
(or a different rule based on imitation) in some circumstances, but NS’s (or the best-response dynamics
discussed in Section 4) in others. Huck \emph{et al.} (1999) present an experiment which supports a similar
idea in a Cournot market. We are grateful to a referee for pointing this out.} Beyond that, an important difference is that imitation requires much
less sophistication than finding a best reply.
Adapted to the insurance market, NS’s model predicts that whenever RS’s competitive equilibrium exists, the RS contracts are the unique long-run outcome; in case of nonexistence, the market has multiple long-run outcomes including the RS contracts and the Wilson pooling contract. The multiplicity is not as severe as in our model because of NS’s zero-profit condition. Still, our results with global experimentation are similar to NS’s. This may seem remarkable, given that the details of the dynamics are so different.

6. CONCLUSION

We have tackled the equilibrium nonexistence problem in Rothschild and Stiglitz’s insurance market in a dynamic context with boundedly rational firms. The behavioral rule underlying our dynamics is based on imitation of profitable contracts, withdrawal of loss-making contracts, and local experimentation. We show that the RS candidate competitive equilibrium is the unique long-run market outcome in that case. This result relies on both, imitation and local experimentation; it would break down if either non-local experiments were as likely as local experiments, or firms were able to find best replies. However, we believe that imitation and local experimentation are plausible for firms operating in real markets with incomplete information. It is unrealistic to assume that firms always have enough information and computation capability to find best replies. It is also more realistic to assume that an experimenting firm makes a local innovation rather than trying out something completely novel.

\[x = I_{\delta_l}, \quad x^* = I_{\delta_l}^{hl}, \quad \text{NS's Proposition 2 and Lemma 3 imply that the insurance market has a unique recurrent set, the states of which are the long-run states. Hence, the RS contracts are the unique long-run outcome if and only if RS's competitive equilibrium exists (see NS, Proposition 3). The Wilson pooling contract is never the unique long-run outcome (see NS, Proposition 4, [4.2](a)). In case of nonexistence of RS's competitive equilibrium, the Wilson pooling contract is a long-run outcome, but not the unique one (see NS, Proposition 5). NS do not prove that the RS outcome is also a long-run outcome in the nonexistence case. Here is a sketch of a proof. It is sufficient to construct a sequence of isolated mutations from the Wilson outcome to the RS outcome. The sequence starts with mutations which increase the premia of all contracts, except the Wilson pooling contract, such that these contracts make zero profits when bought by the high risks only; at this point, the high risks’s RS contract is already offered, but is idle. Then, starting from the Wilson pooling contract, a sequence of cream-skimming mutations makes the low risks’s RS contract appear, and the risk types separate.]\]

\[21\text{To see this, assume that there exists a fine grid of indemnities, including, in NS's notation}, \quad \pi = I_{\delta}^\delta, \quad x = I_{\delta}^l, \quad \text{and} \quad x^* = I_{\delta}^{hl}. \quad \text{NS's Proposition 2 and Lemma 3 imply that the insurance market has a unique recurrent set, the states of which are the long-run states. Hence, the RS contracts are the unique long-run outcome if and only if RS's competitive equilibrium exists (see NS, Proposition 3). The Wilson pooling contract is never the unique long-run outcome (see NS, Proposition 4, [4.2](a)). In case of nonexistence of RS's competitive equilibrium, the Wilson pooling contract is a long-run outcome, but not the unique one (see NS, Proposition 5). NS do not prove that the RS outcome is also a long-run outcome in the nonexistence case. Here is a sketch of a proof. It is sufficient to construct a sequence of isolated mutations from the Wilson outcome to the RS outcome. The sequence starts with mutations which increase the premia of all contracts, except the Wilson pooling contract, such that these contracts make zero profits when bought by the high risks only; at this point, the high risks’s RS contract is already offered, but is idle. Then, starting from the Wilson pooling contract, a sequence of cream-skimming mutations makes the low risks’s RS contract appear, and the risk types separate.}\]

\[22\text{In particular, our cream-skimming dynamics differ from NS’s. Suppose the current active contract is the Wilson pooling contract. It is true that in both models, a cream-skimming mutation/experiment will attract both risk types after one period (in our model, the previous contract is simply withdrawn, while in NS’s model, the premium of the previous contract is increased so that it becomes unattractive even for the high risks). Now making losses, the cream-skimming contract is then withdrawn according to our model, so that the dead-market state is reached unless the previous state included idle contracts. In NS’s model, however, the premium of the cream-skimming contract is increased, so that the contract makes zero profits as a pooling contract. Hence, in their model another cream-skimming mutation may occur, and the RS outcome can be reached by a sequence of such isolated mutations.}\]
The main contribution of our paper is that it offers a dynamic analysis of Rothschild and Stiglitz’s insurance market with boundedly rational firms, thereby overcoming the problem of nonexistence of a prediction.

APPENDIX

Here we prove Proposition 3.1. The analysis of the market dynamics is done in Lemmata A.3–A.6. To prepare that, we provide the technical Lemmata A.1 and A.2. Some notation is introduced first. We think of individuals’ indifference curves as functions that assign premia to indemnities. Given any contract $c\in\mathbb{R}^2$, the slope of type $i$’s ($i = l, h$) indifference curve at $c$ is denoted $s_i(c) < 1$. Continuity of $u$ implies that $s_i$ reaches a maximum value $\bar{s}_i < 1$ in $[0, \bar{I}] \times [0, \bar{P}]$. Moreover, because $u''(\cdot)$ is continuous, there exist a finite upper bound $-\bar{s}_1 < 0$ and a finite lower bound $-\bar{s}_2 < 0$ for the second derivative of the low risks’ indifference curve at any contract in $[0, \bar{I}] \times [0, \bar{P}]$.

The following sets will play a central role throughout the proof. For all $\delta > 0$, and all $c \in \Gamma^\delta$, let

$$ U_l^\delta(c) = \{c' \in \mathcal{N}_l^\delta(c) \mid U_h(c') > U_h(c), \, V(c', \pi_h) > 0\}, $$

$$ U_h^\delta(c) = \{c' \in \mathcal{N}_h^\delta(c) \mid U_l(c') > U_l(c), \, V(c', \pi_l) > 0\}, $$

$$ U_r^\delta(c) = \{c' \in \mathcal{N}_r^\delta(c) \mid U_l(c') > U_l(c), \, V(c', \pi_l) > 0, \, U_h(c') < U_h(c_l')\}. $$

Lemma A.1 provides “local” characterizations of $c_l^\delta, c_h^\delta$ and $c_r^\delta$.

**Lemma A.1.** Let $r > 0$. Then, there exists $\bar{\delta} > 0$ such that (i), (ii), and (iii), hold for all $0 < \delta < \bar{\delta}$ and all $c \in \Gamma^\delta$.

(i) $(V(c, \pi_h) > 0$ and $U_l^\delta(c) = \emptyset) \iff c = c_l^\delta$.

(ii) $(V(c, \pi_l) > 0$ and $U_h^\delta(c) = \emptyset) \iff c = c_h^\delta$.

(iii) $(V(c, \pi_l) > 0$ and $U_h(c) < U_h(c_h^\delta)$ and $U_r^\delta(c) = \emptyset) \iff c = c_r^\delta$.

**Proof.** We only prove (i) (the proofs of (ii) and (iii) are similar and hence omitted). The “$\iff$”-part of (i) is obvious, for all $\delta > 0$. To prove “$\Rightarrow$” in (i), consider $I_1, I_2, P_3 \in \mathbb{R}_+$ such that $I_1 < L < I_2 < \bar{I}$ and $\pi_h I_2 < P_3 < \bar{P}$. Define sets of contracts

$$ M_1 = \{(I, P) \in \mathbb{R}_+^2 \mid I < I_1, \, \pi_h I < P \leq P_3\}, $$

$$ M_2 = \{(I, P) \in \mathbb{R}_+^2 \mid I_1 < I \leq \bar{I}, \, \pi_h I < P\}, $$

$$ M_3 = \{(I, P) \in \mathbb{R}_+^2 \mid I \leq I_2, \, P_3 < P \leq \bar{P}\}. $$
In fact, because $c^R_h = (L, \pi_h L)$, we can choose $I_1, I_2, P_3$ such that\footnote{For all $r > 0$ and $c \in \mathbb{R}^2$, the open $|| \cdot ||$-ball with center $c$ and radius $r$ is denoted $B_r(c) = \{ c' \in \mathbb{R}^2 | ||c' - c|| < r \}$.}

$$\forall c \in \mathbb{R}^2_+, V(c, \pi_h) > 0 : \ c \in B_{r/2}(c^{RS}_h) \cup M_1 \cup M_2 \cup M_3.$$

Together with $c^R_h \rightarrow c^R_h$ this implies that there exists $\tilde{\delta}_0 > 0$ such that

$$\forall 0 < \delta < \tilde{\delta}_0, c \in \mathbb{R}^2_+, V(c, \pi_h) > 0 : \ c \in B_r(c^R_h) \cup M_1 \cup M_2 \cup M_3. \ (A.1)$$

Next we show for $k = 1, 2, 3$ the existence of a $\delta_k > 0$ such that

$$\forall 0 < \delta < \delta_k, c \in \mathbb{R}^2_+ : \ c \in B_r(c^R_h) \cup M_1 \cup M_2 \cup M_3. \ (A.2)$$

We start with $k = 1$. Choose $I'_1$ such that $I_1 < I'_1 < L$, and $P'_3$ such that $P_3 < P'_3 < \overline{P}$. Define

$$M'_1 \equiv \{ (I, P) \in \mathbb{R}^2_+ | I < I'_1, \pi_h IP \leq P'_3 \} \supseteq M_1.$$

Let $s = \inf_{c \in M'_1} \left( s_h(c) < \pi_h < 1 \right)$ be the infimum slope of all $h$-indifference curves through contracts in $M'_1$. Because $I'_1 < L$, we have $s > \pi_h$.

For all $d > 0$ let

$$\delta_1(d) = \frac{d(s - \pi_h)}{2} > 0.$$

For all $c = (I, P) \in \mathbb{R}^2_+$ and $d > 0$, we define an open square,

$$Q(c, d) = [I + d - \delta_1(d), I + d] \times [P + \pi_h d, P + \pi_h d + \delta_1(d)],$$

a cone section

$$N(c, d) = \{ (I', P') \in \mathbb{R}^2_+ | I < I'_1 < I + d, \pi_h < \frac{P' - P}{I' - I} < s \},$$

and a rectangle

$$R(c, d) = [I, I + d] \times [P, P + sd].$$

A calculation shows

$$Q(c, d) \subseteq N(c, d) \subseteq R(c, d). \quad (A.3)$$

Defining $\tilde{d} = \min \{ I'_1 - I_1, (P'_3 - P_3)/s \}$, we get

$$\forall 0 < d < \tilde{d}, \ c \in M_1 : \ R(c, d) \subseteq M'_1. \quad (A.4)$$
In particular, (A.3) and (A.4) imply
\[ Q(c, d) \subseteq R(c, d) \subseteq M'_1 \subseteq [0, \overline{T}] \times [0, \overline{T}], \]
and hence (using that \( Q(c, d) \) has the side length \( \delta_1(d) \)),
\[ \forall 0 < d < \overline{d}, \quad 0 < \delta < \delta_1(d), \quad c \in M_1 : \ Q(c, d) \cap \Gamma^d \neq \emptyset. \tag{A.5} \]
From (A.3) and (A.4) we get \( N(c, d) \subseteq M'_1 \). Together with the definition of \( \varepsilon \) this yields
\[ N(c, d) \subseteq \{ c' \in \mathbb{R}^2_+ | U_h(c') > U_h(c), \ V(c', \pi_h) > 0 \}. \tag{A.6} \]
Because all norms in \( \mathbb{R}^2 \) are equivalent, there exists \( 0 < \tilde{d} < \overline{d} \) such that
\[ \forall c \in \mathbb{R}^2_+ : \ R(c, \tilde{d}) \subseteq B_r(c). \tag{A.7} \]
Defining \( \delta_1 = \delta_1(\tilde{d}) \), (A.3), (A.5), (A.6), and (A.7) imply that
\[ \forall 0 < \delta < \delta_1, \quad c \in M_1 : \ Q(c, \tilde{d}) \cap U^\delta_h(c) \neq \emptyset. \]
From this, (A.2) is immediate for \( k = 1 \).
For \( k = 2 \), the proof of (A.2) is obtained by a similar geometric construction. We omit the details. We now show (A.2) for \( k = 3 \). Because all norms in \( \mathbb{R}^2 \) are equivalent, there exists \( \delta_3 > 0 \) such that
\[ \forall c = (I, P) \in \mathbb{R}^2_+ : \ (I, P - \overline{T}_3) \in B_r(c). \]
Therefore, defining \( \overline{T}_3 = \min \{ \overline{T}_3, P_3 - \pi_h I_2 \} > 0 \) we get
\[ \forall c = (I, P) \in M_3 \cap \Gamma^\delta, \quad 0 < \delta < \overline{T}_3 : \ (I, P - \delta) \in U^\delta_h(c). \]
This implies (A.2) for \( k = 3 \). Now define \( \overline{T} = \min \{ \delta_0, \ldots, \delta_3 \} \).
To complete the proof of “\( \Rightarrow \)” in (i), let \( 0 < \delta < \overline{T}, \ c \in \Gamma^\delta, \ V(c, \pi_h) > 0 \) and \( U^\delta_h(c) = \emptyset \). Then, (A.1) and (A.2) for \( k = 1, 2, 3 \) together imply that \( c \in B_r(e^\delta_h) \).
Consequently, \( c^h \in N^\delta_h(c) \), implying \( c = c^h \).

Lemma A.2 considers contracts such that the indemnity is at least \( \nu_1 > 0 \) and the profit with a low risk is at least \( \nu_2 > 0 \). The neighborhood of any such contract is shown to contain a contract which attracts only the low risks and makes profits if \( \delta \) is sufficiently small.
LEMMA A.2. Let \( r > 0, \nu_1 > 0, \nu_2 > 0, \) and
\[
K \equiv \{(I, P) \in \mathbb{R}^2 \mid \nu_1 \leq I \leq \bar{I}, \pi_1 \cdot I + \nu_2 \leq P \leq \bar{P}\}.
\]
There exists \( \delta > 0 \) such that for all \( 0 < \delta < \bar{\delta} \) and all \( k \in K \cap \Gamma^\delta \), we have
\[
\emptyset \neq C^\delta(k) \equiv \{c' \in \mathbb{N}^R \mid U_h(c') < U_h(k), U_l(c') > U_l(k), V(c', \pi_l) > 0\}.
\]

Proof. We define
\[
\Delta s = \min_{c \in [0, I] \times [0, P]} s_l(c) - s_h(c) > 0.
\]
For all
\[
0 < d < \bar{d} \equiv \min\{\frac{\Delta s}{2s_2}, \frac{\nu_1}{\bar{s}_h}, \frac{\nu_2}{s_h}\}, \tag{A.8}
\]
let \( \bar{d}(d) = (d\Delta s)/4 > 0 \). For all \( c = (I, P) \in \mathbb{R}_+^2 \) and \( 0 < d < \bar{d} \), we define an open square,
\[
Q(c, d) = \{I - d, I - d + \bar{d}(d)\} \times \{P - (s\bar{h}(c) - \frac{\Delta s}{2})d - \bar{d}(d), P - (s\bar{h}(c) - \frac{\Delta s}{2})d\},
\]
a cone section
\[
N(c, d) = \{(I', P') \in \mathbb{R}_+^2 \mid I - d < I' < I, s\bar{h}(c) - \frac{\Delta s}{2} < \frac{P - P'}{I - I'} < s\bar{h}(c)\},
\]
and a rectangle
\[
R(c, d) = \{I - d, I\} \times \{P - d\bar{s}_h, P\}.
\]
A calculation shows
\[
Q(c, d) \subseteq N(c, d) \subseteq R(c, d). \tag{A.9}
\]
Moreover, using \( d < \min\{\nu_1, \nu_2/\bar{s}_h\} \) from (A.8), we get
\[
\forall k \in K : R(k, d) \subseteq \{c' \in [0, I] \times [0, \bar{P}] \mid V(c', \pi_l) > 0\}. \tag{A.10}
\]
In particular, (A.9) and (A.10), with \( c = k \), imply
\[
Q(k, d) \subseteq R(k, d) \subseteq [0, I] \times [0, \bar{P}],
\]
and hence (using that $Q(k, d)$ has the side length $\delta(d)$),
\[ \forall 0 < d < \delta', 0 < \delta < \delta(d), k \in K : Q(k, d) \cap \Gamma^\delta \neq \emptyset. \]  
(A.11)

Moreover, a calculation yields that\footnote{To see (A.12), let $l' \mapsto f(l')$ denote the $l'$-indifference curve through the contract $k$. It is sufficient to show the following: if $l' \in [I - d, I]$ then $f(l') - (s_I(k) - \Delta s/2)(I' - I) + P' > 0$. Using $f(I) = P$; and $f'(I) = s_l(k)$, and $s_l(k) - \Delta s/2 \geq s_l(k) + \Delta s/2$, Taylor’s theorem (expansion with center $I$, first order approximation with second order error term) yields $f(l') \geq (I - I')(\Delta s/2 - I - I'(-f''(I'))))$ with some $I^* \in [I', 2I - I']$. To get the desired inequality $f(l') > 0$, one now uses $-f''(I^*) \leq \delta_2$, and $I - I' < d$, and the assumption $d < \Delta s/(2\delta_2)$ from (A.8).}
\[ \forall k \in K : N(k, d) \subseteq \{ c' \in \mathbb{R}^2_+ \mid U_h(c') < U_h(k), U_l(c') > U_l(k)\} \]  
(A.12)

Because all norms in $\mathbb{R}^2$ are equivalent, there exists $0 < \tilde{\delta} < \delta$ such that
\[ \forall c \in \mathbb{R}^2_+ : R(c, \tilde{\delta}) \subseteq B_r(c). \]  
(A.13)

Defining $\delta = \delta(\tilde{\delta})$, (A.9), (A.10), (A.11), (A.12), and (A.13), with $c = k$, imply that
\[ \forall 0 < \delta < \delta, k \in K \cap \Gamma^\delta : Q(k, \tilde{\delta}) \cap C^\delta(k) \neq \emptyset, \]
which completes the proof. \[ \Box \]

Now we turn to the analysis of the dynamics. For any two states $s, s' \in \Omega^\delta$ the resistance $r(s, s')$ is the minimum number of experiments in any finite sequence of transitions that brings the process from $s$ to $s'$ (cf. Young, 1993). A nonempty set $Q \subseteq \Omega^\delta$ is called absorbing if $Q$ is minimal with respect to the property that for all $s \in Q$, $s' \notin Q$ we have $r(s, s') \neq 0$. A state $s$ is called absorbing if $\{s\}$ is an absorbing set. Proposition 2.1 shows that every absorbing set is a singleton, and characterizes the absorbing states.

A sequence of isolated experiments from an absorbing state $s$ to an absorbing state $s'$ is defined as a finite sequence of absorbing states $s_1, \ldots, s_q$ ($q \geq 1$) such that $s_1 = s$, $s_q = s'$, and $r(s_i, s_{i+1}) = 1$ for $i = 1, \ldots, q - 1$. If a sequence of isolated experiments from $s$ to $s'$ exists, we write $s \Rightarrow s'$. We write $s \iff s'$ if $s \Rightarrow s'$ and $s' \Rightarrow s$. An equivalence class $R$ of $\iff$ is called a locally stable component if there do not exist absorbing states $s \in R$ and $s' \notin R$ with $r(s, s') = 1$. By Nöldke and Samuelson (1993, Proposition 1) the set of long-run states is the union of some of the locally stable components.

Lemma A.3 shows that if a locally stable component includes one RS state then it includes all. Lemma A.4 shows that any locally stable component must include
an RS state. These two results imply (a). Lemma A.5 implies (b), and Lemma A.6 implies (c).

Let $R^\delta$ be the set of RS states.

**Lemma A.3.** If $s, s' \in R^\delta$ then $s \Rightarrow s'$.

**Proof.** By withdrawal-experiments, $s \Rightarrow v \equiv (S, \ldots, S)$, where $S = \{c_1^\delta, c_h^\delta\}$.

It remains to be shown that $v \Rightarrow s'$.

Fix some $(I, P) \in S$, and let $(I', P')$ be an idle contract $(I', P')$ offered by firm $j$ in state $s'$. Note that $P' > P$ or $I' > I$. Assume $j = 1$, $P' \geq P$, and $I' \geq I$; all other cases are omitted because they can be treated analogously.

Consider the following sequence of contracts (with $q$ chosen appropriately):

$$(c_1, \ldots, c_q) \equiv ((I, P), (I, P + \delta), (I, P + 2\delta), \ldots, (I, P'), (I + \delta, P'), (I + 2\delta, P'), \ldots, (I', P')).$$

For $k = 1, \ldots, q$, let $s_k$ be the state in which firm 1’s menu is $\{c_1, \ldots, c_k\} \cup S$, and each other firm’s menu is $S$.

All contracts $\{c_2, \ldots, c_{q-1}\}$ are idle in all states $s_1, \ldots, s_q$ (note that $c_1$ or $c_q$ are preferred). Therefore, the states $s_1, \ldots, s_q$ are absorbing. In state $s_k$ ($k = 1, \ldots, q - 1$), the contract $c_k$ belongs to the menu of firm 1. Hence, there exists an experiment which adds contract $c_k + 1$ to the menu of firm 1, implying

$$v = s_1 \Rightarrow \ldots \Rightarrow s_q.$$ 

Repeating these arguments, one sees that there exists a sequence of isolated experiments from $v$ to $v'$, where $v'$ is an absorbing state such that (i) the set of profitable contracts is $S$; and (ii) if any firm offers any idle contract in state $s'$ this firm offers the same idle contract also in $v'$. Using withdrawal-experiments with idle contracts, it is clear that a sequence of isolated experiments from $v'$ to $s'$ exists. In summary, we have shown $s \Rightarrow v \Rightarrow v' \Rightarrow s'$.

**Lemma A.4.** Let $\delta > 0$. For all sufficiently small $\delta > 0$, the following holds. For all absorbing states $s \in \Gamma^\delta$ there exists $w \in R^\delta$ such that $s \Rightarrow w$.

**Proof.** Let $[s]_i$ denote the contract bought by type $i$ in state $s$ (and $[s]_i = (0, 0)$ if type $i$ abstains from the market). Each absorbing state belongs to one of the following sets.

$$\mathcal{A} = \{s \in \Gamma^\delta \mid s \text{ is absorbing, } [s]_h = c_h^\delta\},$$

$$\mathcal{B} = \{s \in \Gamma^\delta \mid s \text{ is absorbing, } [s]_h \neq c_h^\delta, \mathbb{V}(\pi_h) > 0\},$$
Lemma A.1(i). By construction, \( c \) imitate the experimenting firm. After \( c \) introduced, it attracts the low risks and only these, and thus all competitors will be introduced by successive isolated experiments of any firm. When a contract is proof of Lemma A.1; note that \( \exists \) experiments which removes all idle contracts from the market. Hence, w.l.o.g., \( \exists \).

Next we prove \( A \). (i) \( \forall s \in A \exists w \in R^5 : s \Rightarrow w. \)

B. \( \forall s \in B \exists s' : s \Rightarrow s', (s' \in A \text{ or } (s' \in B, U_h([s']_h) > U_h([s]_h))). \)

C. \( \forall s \in C \exists s' : s \Rightarrow s', (s' \in B \cup D \text{ or } (s' \in C, U_l([s']_l) > U_l([s]_l))). \)

D. \( \forall s \in D \exists s' \in A \cup B : s \Rightarrow s'. \)

To prove A, we construct a finite sequence of contracts \( (c_1, c_2, \ldots, c_q) (q \geq 1). \) If \( [s]_l \neq (0, 0) \) then let \( c_1 = [s]_l; \) otherwise, for all sufficiently small \( \delta \), there exists \( c_1 \in N^5([0, 0]) \) such that \( V(c_1, \pi_l) > 0, U_I(c_1) > U_I((0, 0)), \) and \( U_h(c_1) < U_h(c_1) \) (the techniques used to prove this fact are similar to those applied in the proof of Lemma A.1; note that \( s l([0, 0] > \pi_l). \) If \( c_1 = c^2_1 \) then set \( q = 1 \); otherwise, \( U_l(c_1) \neq \emptyset \) by Lemma A.1 (iii). Hence, if \( c_1 \neq c^2_1 \) then there exists \( c_2 \in U^h(c_1). \) If \( c_2 = c^2_1 \) then set \( q = 2; \) otherwise, \( U^h(c_2) \neq \emptyset \), and the construction is completed inductively (it ends with a finite \( q \) because \( U_l(c_1) < U_I(c_2) < \ldots ). \)

Starting in state \( s \), the contracts \( c_2, \ldots, c_q \) (resp., \( c_1, \ldots, c_q \) if \([s]_l = (0, 0)\)) can be introduced by successive isolated experiments of any firm. When a contract is introduced, it attracts the low risks and only these, and thus all competitors will imitate the experimenting firm. After \( c_q \) is imitated, an RS state is reached.

Next we prove B. In state \( s \), there exists an experiment \( c_h' \in U^h_h([s]_h) \) by Lemma A.1(i). By construction, \( c_h' \) is profitable. By imitation of \( c_h' \), the market reaches a new absorbing state \( s' \) with

\[ [s']_h = c_h', [s']_l \in \{[s]_l, c_h'\}, U_h([s']_h) > U_h([s]_h). \]

Either \( c_h' = c^2_h \) (then \( s' \in A \)) or \( c_h' \neq c^2_h \) (then \( s' \in B \)).

To prove C, denote \( c = [s]_l. \) There exists a sequence of isolated withdrawal-experiments which removes all idle contracts from the market. Hence, w.l.o.g., \( s = \{[c], \ldots, [c]\}. \) First consider the case \( c \neq c^2_h \). In state \( s \), there exists an experiment \( c' \in U^h_h(c) \) by Lemma A.1(iii). Because \( c' \) attracts the low risks, it is profitable. By imitation of \( c' \), the market reaches a state \( v \) with \([v]_l = c' \) and either \([v]_h = c \) (then \( c \) will be withdrawn because \( V(c, \pi_h) < 0 \), and then an absorbing
state $s' \in B \cup C$ is reached) or $[v]_{h} = c'$ (then $v \in C$, and thus we can set $s' = v$).

In both cases, $U_{t}([s']) > U_{t}([s])$.

Secondly, consider the case $c = c_{hl}$. Recalling the notation $c_{hl} = (I_{hl}, P_{hl})$ for the Wilson pooling contract, we define $\nu_{1} = I_{hl}/2$, $\nu_{2} = (P_{hl} - \pi_{l}I_{hl})/2$, and $k = c_{hl}$. The assumptions of Lemma A.2 are fulfilled for all sufficiently small $\delta$: we have $I_{hl} > 0$ (otherwise $c_{hl} = (0, 0)$, thus $U_{t}(c_{hl}) < U_{t}((0, 0))$ by the definitions of $c_{hl}$ and $c_{hl}'$, which contradicts that the low risks buy $c = c_{hl}'$, and $c_{hl}' \in K$ (because $c_{hl} \rightarrow c_{hl}$ and $K$ contains an open neighborhood of $c_{hl}$). By Lemma A.2, there exists (in state $s$) an experimental contract $c' \in C^{\delta}(c_{hl})$ which attracts the low risks only and is profitable, while $c_{hl}'$ is then bought by the high risks and makes losses. Hence, one period after the experiment $c'$ is imitated and $c_{hl}'$ is withdrawn. Consequently, two periods after the experiment $c'$ is bought by everybody and thus makes losses. Thus, $c'$ is withdrawn and the market has reached the dead-market state $(\emptyset, \ldots, \emptyset) \in D$.

We now prove $D$. If $\delta > 0$ is sufficiently small, there exist $\nu_{1} > 0$ and $\nu_{2} > 0$ such that, with $K$ defined as in Lemma A.2, there exists $k \in K \cap N_{\delta}(0, 0)$ with $V(k, \pi_{h}) > 0$ and $U_{h}(k) > U_{hl}(0, 0)$ (the techniques used to prove this fact are similar to those applied in the proof of Lemma A.1; note that $s_{l}h(0, 0) > \pi_{h}$). Hence, by Lemma A.2 there exists a contract $c_{l} \in C^{\delta}(k)$.

In state $s$, there exists an experiment which adds $k$ to some firm’s menu. By construction, $k$ makes profits and is imitated by all firms. In the absorbing state reached then, there exists an experiment which adds $c_{l}$ to some firm’s menu, and imitation leads to an absorbing state $s'$ with $[s']_{l} = c_{l}$ and $[s']_{h} = k$. Thus, $s' \in A \cup B$.

**Lemma A.5.** Let $0 < r < r_{0}$. Then, $R^{\delta}$ is a locally stable component for all sufficiently small $\delta > 0$.

**Proof.** By Lemma A.3, it is sufficient to show the following.

Suppose, at the beginning of period $t$ the current state is some $s = s(t) \in R^{\delta}$, and in period $t$ some firm $j$ makes an experiment which leads to some state $s(t + 1) \notin R^{\delta}$, and no experiment occurs from period $t + 1$ onwards. Then there exists $t' > t + 1$ such that $s(t') \in R^{\delta}$.

First, consider the case where firm $j$ makes a withdrawal-experiment. None of firm $j$’s competitors will change its menu because $c_{hl}^{d}$ and $c_{hl}^{u}$ remain profitable in period $t + 1$. Thus, firm $j$ will imitate all the RS contracts it had withdrawn, and we have $s(t + 2) \in R^{\delta}$.

Second, suppose firm $j$ makes an innovation-experiment. By Proposition 2.1, in some period $t' > t + 1$ the market reaches an absorbing state $s(t')$ with $s(t') \subseteq s(t + 1)$. It remains to be shown that (i) $\{c_{hl}^{d}, c_{hl}^{u}\} \subseteq S_{j}(t')$, and (ii) all contracts in $N \equiv s(t') \setminus \{c_{hl}^{d}, c_{hl}^{u}\}$ are idle.

We have $c_{hl}^{d} \in S_{j}(t'')$ for all $t'' = t, \ldots, t'$ because $c_{hl}^{d}$ can never make losses, no matter who gets attracted. Therefore, $c_{hl}^{d}$ does not attract the high risks in any
period $t, \ldots t'$, implying that it cannot make losses in any of these periods either. This shows $c^h_l \in S_j(t')$ and thus (i) holds.

To prove (ii), suppose some $c \in N$ is not idle. Note that $c$ is profitable because $s(t')$ is absorbing. Also, $c$ is not bought by the high risks only (otherwise $U_h(c) > U_h(c^h_l)$ and $V(c, \pi_h) > 0$, a contradiction to the definition of $c^h_l$). Neither is $c$ bought by everybody (otherwise $c \in \mathcal{P}^h$, a contradiction to Remark 2.2 by assumption $r < r_0$). Hence, $c$ is bought by the low risks only, and

$$U_h(c) > U_h(c^h_l)$$

by definition of $c^h_l$. Therefore, there exists $c' \in N$ with

$$U_h(c') > U_h(c),$$

and $c'$ is bought by the high risks only. From (A.14) and (A.15) we get that $U_h(c') > U_h(c^h_l)$ and thus $c'$ makes losses by definition of $c^h_l$. But this contradicts that $s(t')$ is absorbing!

**Lemma A.6.** Let $\mathcal{P}^h \neq \emptyset$ and $r > \max_{c,c' \in \mathcal{P}^h} ||c - c'||$. Then, for all sufficiently small $\delta > 0$, the following holds. If $s \in \mathcal{R}^h$ and $s'$ is absorbing then $s \Rightarrow s'$.

**Proof.** Given the methods introduced in the previous lemmata, the proof is straightforward from the explanations on p. 16.

**REFERENCES**